Celestial Dynamics and Astrometry in an Expanding Universe

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Content of the lecture:

1. The ultimate goal
2. Two spacetimes
3. Two physical principles
4. What time is physical?
5. Clocks
6. Newtonian limit in cosmology
7. Light time inhomogeneity
8. Doppler tracking and range measurements
9. Can the Hubble expansion be measurable in the solar system?

The ultimate goal of celestial mechanics and astrometry is to study the fundamental structure of spacetime geometry underlying the world.

The truth of the geometry of space and time depends on the truth of axioms and, therefore, the question is whether the axioms are true.

There are two methods of testing the axioms of spacetime geometry:

- *Extrinsic* – look at the world "from external" dimensions (high-energy physics, LHC)

- *Intrinsic* – measure the internal relationships between geometrical entities of the spacetime manifold

- RCMA in the solar system was built under assumption that the background geometry is a static Minkowski spacetime

$$d\sigma^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

↑

TCB

- On the other hand, the background geometry in cosmology is the RW spacetime ("expanding space")

$$ds^2 = -c^2dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

↑

Physical time of the Hubble observers measured by clocks that are synchronized

$$a(t) = 1 + \mathcal{H}t + \tfrac{1}{2}\mathcal{H}'t^2 + \cdots$$

↑

Hubble's parameter $\mathcal{H} = a'/a$

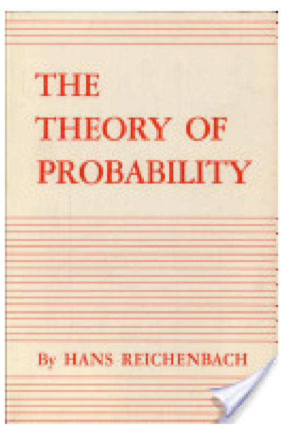
- ✓ Do the two geometries match?
- ✓ At what level?
- ✓ What principle(s) should we use to extrapolate RCMA from the Minkowski world to the expanding universe?
- ✓ Will the Newtonian equations be the same?
- ✓ Should we expect new effects?
- ✓ If, yes, are they observable?
- ✓ Can we find “non-expanding” standards of length in the “expanding space”?

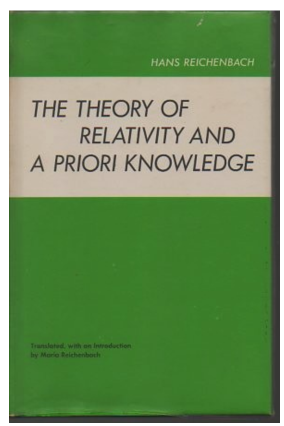
Causality – the driving principle

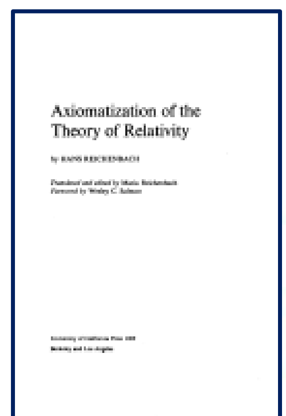
We arbitrarily can choose the geometry, or we arbitrarily can choose the causality; but we cannot choose both.

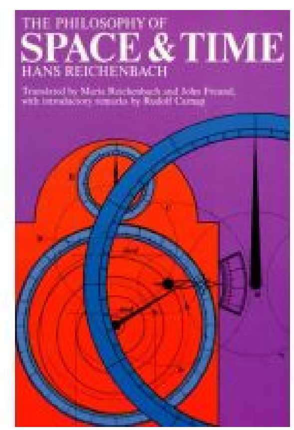
Hans Reichenbach

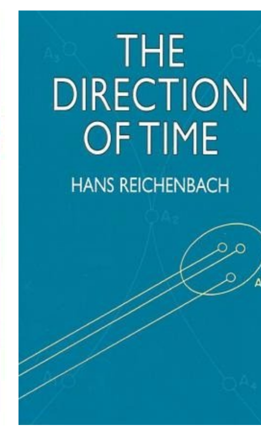
The Philosophy of Space and Time (1928)











Principle of equivalence versus causality principle:

For the RW cosmological spacetime two physical requirements

1) *Local geometry must be Minkowskian,*

$$g_{\alpha\beta}(0) = \eta_{\alpha\beta} = \mathrm{diag}[-1,1,1,1]; \qquad g_{\alpha\beta,\gamma}(0) = 0\,;$$

2) *Local causality is predetermined by the cosmological metric,*

are incompatible in a linear order with respect to the Hubble constant, $\mathcal{H} = a'/a$ **.**

There is no coordinate transformation from the RW metric to the local Minkowski metric that preserves the structure of the physical light cone outside of observer's world line.

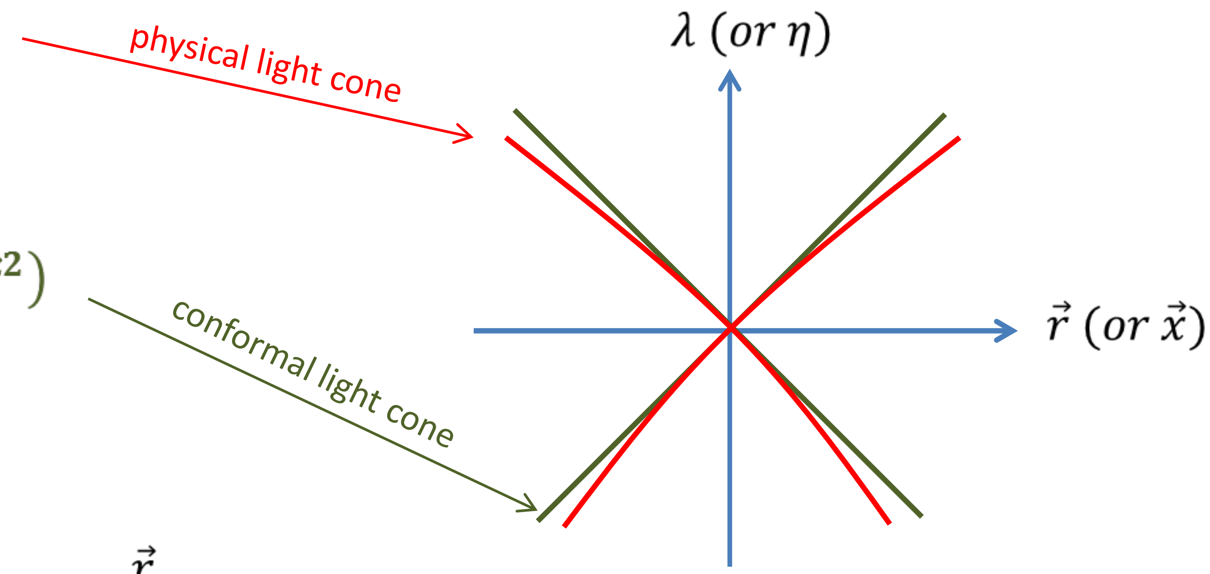
What time is physical?

Lemma: **two conformally equivalent metrics have the same causal structure (identical light cones)**

$$ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^2$$

$$d\eta = \frac{dt}{a(t)}$$

$$ds^2 = a^2(t)\left(-c^2 d\eta^2 + d\vec{x}^2\right)$$



$$ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^2$$

$$t = \lambda - \frac{\mathcal{H}\vec{r}^2}{2c^2} + O(\mathcal{H}^2)\,; \qquad \vec{x} = \frac{\vec{r}}{a(t)} + O(\mathcal{H}^2)$$

$$ds^2 = -c^2 d\lambda^2 + d\vec{r}^2$$

1) **Because the two "green" metrics are conformally equivalent, the coordinate time λ is a causal analogue of the conformal (*non-physical*) time η.**
2) **Physical time is the time of the Hubble observers, t. It is an analogue of TCB.**

Physical Realization of Clocks

- **Atomic Clock**
  - ✓ is the electric charge conserved? **Yes**
  - ✓ is the Coulomb constant, $k = \epsilon_0 / 4\pi$, time-independent? **Yes**
  - ✓ have electron orbitals a constant period (frequency)? **Yes**
- **Gravitational Clock**
  - ✓ are mass, momentum, etc. conserved? **Yes**
  - ✓ is the universal gravitational constant, G , time-independent? **Yes**
  - ✓ have planetary orbits constant periods? **Yes**
- **Light Clock (*light travels between two mirrors*)**
  - ✓ does optical length (range) remain the same? **Yes**
  - ✓ does frequency of light remain the same? **No**

Newtonian Limit of RCMA in cosmology

- **Solve Einstein and Maxwell equations on expanding cosmological manifold in conformal coordinates $(\eta, \vec{x})$. Messy business!**
- **Introduce physical coordinates $(t, \vec{r})$ where $\vec{r} = a(t)\vec{x}$.**
- **Transform equations of motion from the conformal to physical coordinates.**
- **Take the Newtonian limit by making $c \to \infty$ in the equations.**

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{r^3}\vec{r}$$

Newton's law of gravity

$$\frac{d^2\vec{r}}{dt^2} = \frac{k}{m_e}\frac{eQ}{r^3}\vec{r}$$

Coulomb's law of electrostatic

$$\frac{d^2\vec{r}}{dt^2} = \mathcal{H}\frac{d\vec{r}}{dt}$$

Light propagation

Light time inhomogeneity

- Photons accelerate in physical coordinates $(t, \vec{r})$.
- A new time scale (re-parameterization of the light ray)

$$\lambda = a(\eta)\eta = t + \frac{1}{2}\mathcal{H}t^2$$

simplifies the light propagation equation to

$$\frac{d^2\vec{r}}{d\lambda^2} = 0$$

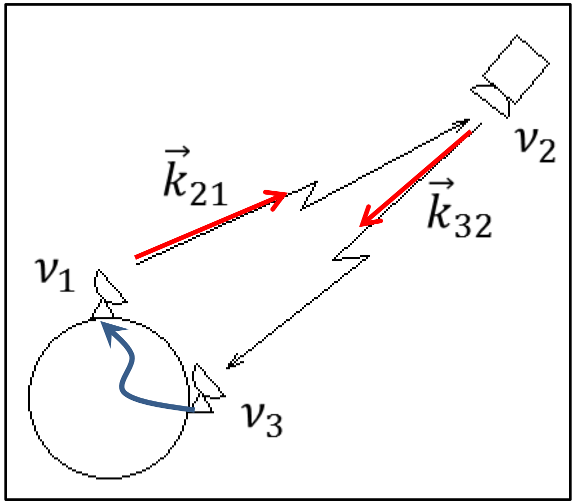
- Trajectory of light in an expanding universe in terms of λ time

$$\vec{r} = \vec{r}_0 + c\,\vec{k}\lambda \qquad \lambda_2 - \lambda_1 = \frac{1}{c}|\vec{r}_2 - \vec{r}_1|$$

- One can check that the quadratic-in-time t inhomogeneity of the parameter λ does not affect ranging distances – they remain the

Doppler tracking

$$\frac{\nu_2}{\nu_1} = \frac{1 + \mathcal{H}t_2 - \vec{k}_{21}\cdot\vec{\beta}_2}{1 + \mathcal{H}t_1 - \vec{k}_{21}\cdot\vec{\beta}_1} \qquad \frac{\nu_3}{\nu_2} = \frac{1 + \mathcal{H}t_3 - \vec{k}_{32}\cdot\vec{\beta}_3}{1 + \mathcal{H}t_2 - \vec{k}_{32}\cdot\vec{\beta}_2}$$



$$\frac{\nu_3}{\nu_1} = \frac{\nu_3}{\nu_2}\frac{\nu_2}{\nu_1} = 1 + 2\mathcal{H}(t_2 - t_1) - 2\vec{k}_{21}\cdot(\vec{\beta}_2 - \vec{\beta}_1)$$

Assuming that $t_3 - t_2 \simeq t_2 - t_1$

Assuming that $\vec{k}_{32} \simeq -\vec{k}_{21}$ and $\vec{\beta}_3 \simeq \vec{\beta}_1$

Integrated Doppler tracking over a long time interval *t (can be for years with current technology!)*

$$\frac{\nu_{2n+1}}{\nu_1} = \frac{\nu_{2n+1}}{\nu_{2n-1}}\frac{\nu_{2n-1}}{\nu_{2n-3}}\cdots\frac{\nu_3}{\nu_1} \Longrightarrow \frac{\nu_{obs}}{\nu_{ref}} = 1 + 2\mathcal{H}t - \frac{2}{c}\int_0^t a_r(t')dt'$$

$$\frac{\nu_{obs}}{\nu_{ref}} = 1 - \frac{2}{c}\int_0^t [a_r(t') - \mathcal{H}c]dt' \Longrightarrow \frac{\nu_{obs} - \nu_{mod}}{\nu_{ref}} = 2\mathcal{H}t$$

blue shift of frequency

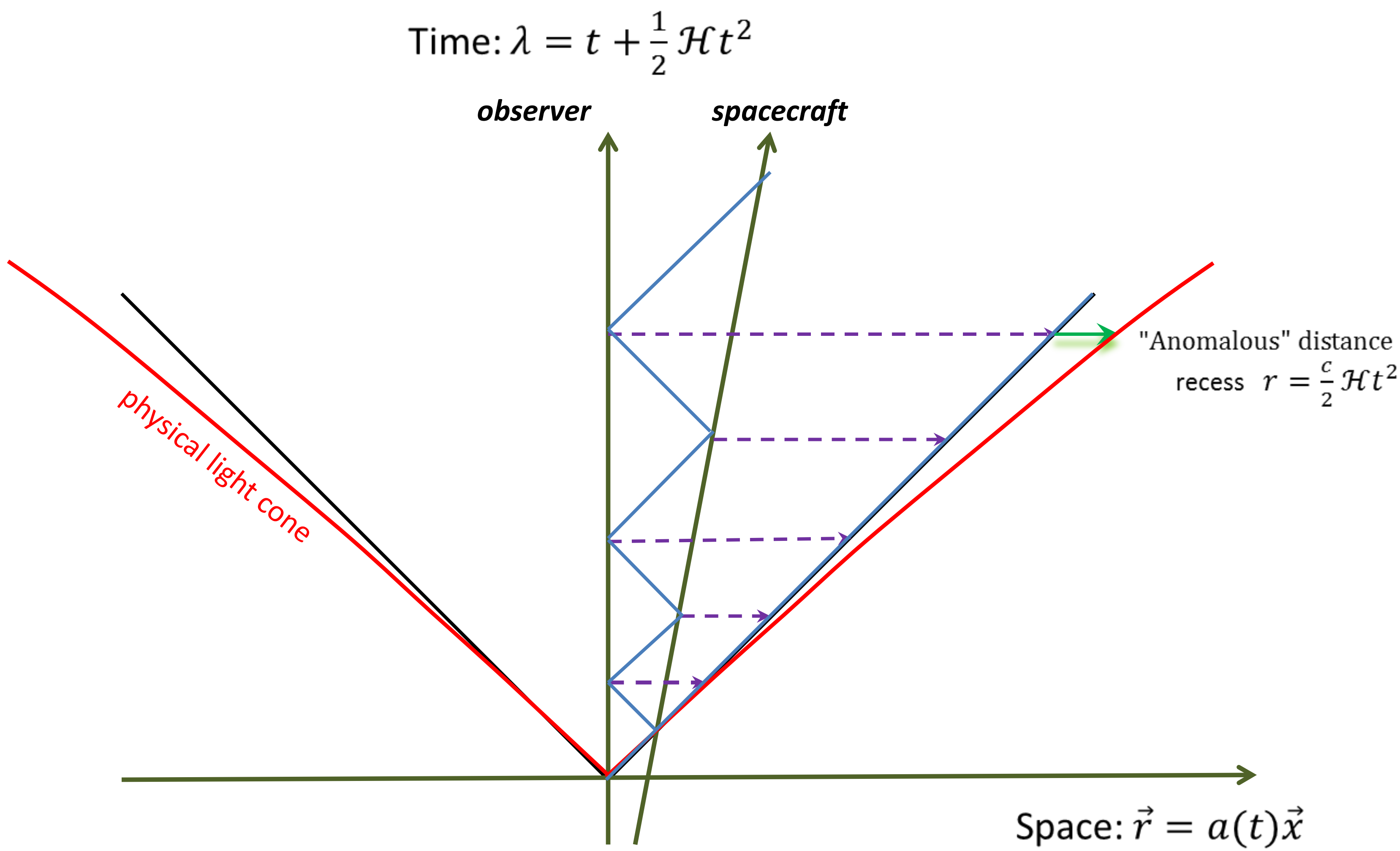
$$a_{spacecraft} = a_{model} - \mathcal{H}c$$

The constant "anomalous acceleration"

$a_{anomaly} = -\mathcal{H}c$ ***directed toward observer.***

Model ephemeris is fully based on ranging.

Minkowski diagram of the integrated Doppler tracking



Is the Hubble expansion observable?

- **Range measurements**

$$c(t_2 - t_1) = |\vec{r}_2 - \vec{r}_1| - \frac{H}{2c}(r_2^2 - r_1^2)$$

The Hubble correction is 10^{-7} *cm* for LLR and 0.1 *cm* for Mars-Earth ranging. Ranging agrees with the new definition of au.

- **Spacecraft Doppler tracking**

$$\frac{\nu_{\rm obs} - \nu_{\rm model}}{\nu_0} = 2Ht$$

- **It is plausible that this equation explains the origin of the "anomalous acceleartion" $a_P = -Hc$ visible as a tiny blue Doppler shift of radio frequency on the top of a red shift due to the (outward) radial velocity of the Pioneer spacecraft.**

- **Theorists tried to find out spacetime "force" acting on spacecraft but could not**

THANK YOU!

More details: http://arxiv.org/abs/1207.3873
Published online: Physical Review D, September 4th