A Generalized Theory of the Figure of the Earth Interior & Application to geoid & H

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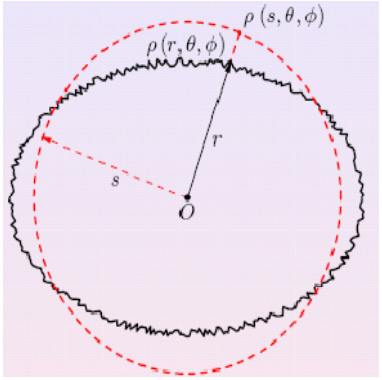

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Outline

- **Background:**
  - **the problems in the traditional theory of the figure of the earth**
  - **The difference of the global dynamical flattening (H) between the theoretical value & observation**
- **Our new theory of the figure of the earth: a generalization**
- **Applications: The direct and indirect contribution of the anti-symmetric crust layer to**
  - **the interior equilibrium figures- Geoid;**
  - **H & MoI**

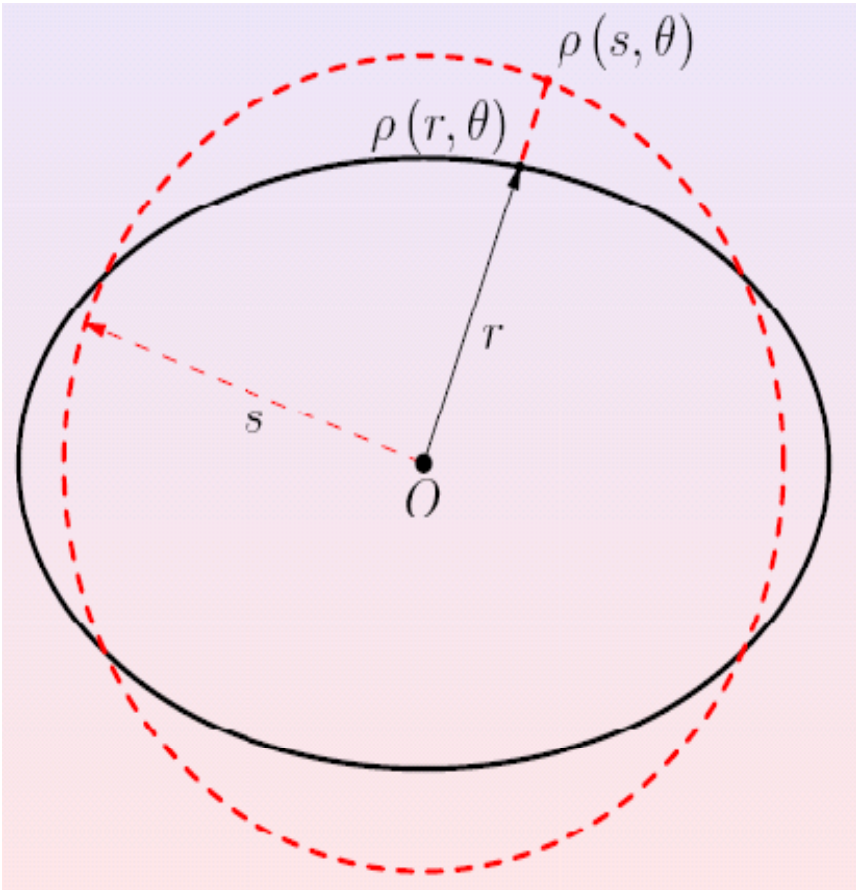
Status & problems (1):

problems in the traditional theory of the figure of the earth

Clairaut (1743) theory (1st order theory)

$$r(s,\theta,\phi) = s[1-\frac{2}{3} fP_2(\cos\theta)]$$

$$\frac{d^2 f}{dq^2} + \frac{6}{q}\frac{\rho}{D}\frac{df}{dq} - \frac{6}{q^2}\left(1-\frac{\rho}{D}\right) f = 0$$



Status & problems (1)

- **Darwin - de Sitter theory (2nd order theory)**

$$r = s\left[1 - f\cos\theta - \left(\frac{3}{8}f^2 + k\right)\sin^2 2\theta\right]$$

$$\beta^2\ddot{\kappa} + 6\frac{\delta}{D}\beta\dot{\kappa} + \left(-20 + 6\frac{\delta}{D}\right)\kappa = 3\left(1 - \frac{\delta}{D}\right)e^2 +$$

$$+ \left(1 - \frac{9}{2}\frac{\delta}{D}\right)\beta e\dot{e} - \frac{1}{4}\left(1 + 9\frac{\delta}{D}\right)\beta^2\dot{e}^2 \quad .$$

Ref: Moritz H., 《The figure of the Earth》

Status & problems (1)

- **Denis (1989) theory (3rd order theory)**

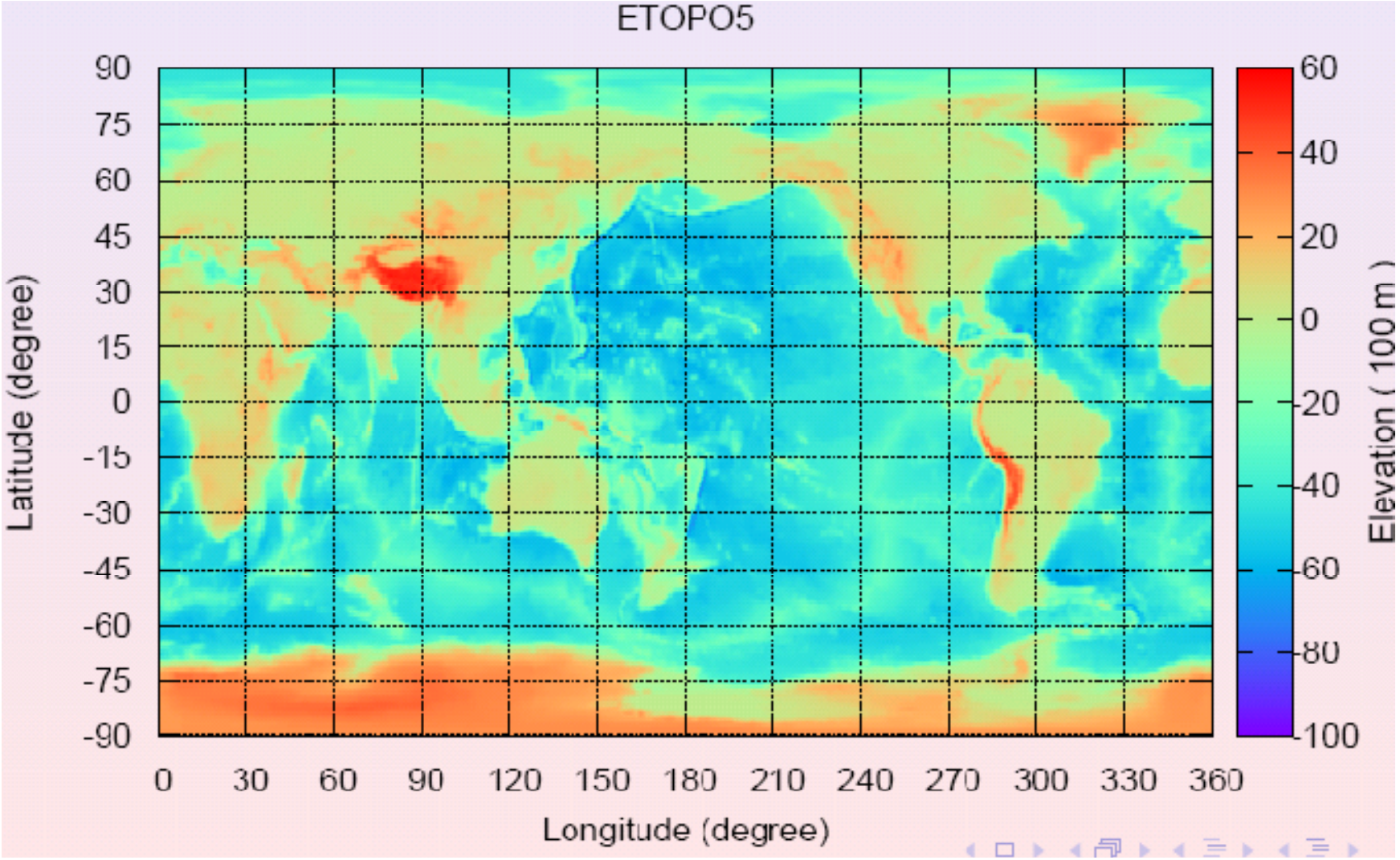
$$r = s\left[1 + s_2 P_2 + s_4 P_4 + s_6 P_6\right]$$

Status & problems (1)

➔All assume that the earth is of rotating symmetric (n=even) and equatorial symmetric (m=0).

But our real earth is obviously NOT!

Our real earth is without any symmetry, but of topography!



Status & problems (2):
problems in the global dynamic flattening (H)

$$H = \frac{C - \frac{A+B}{2}}{C}$$

$$C = \int_v (x^2 + y^2)\, \rho \, dV$$

H is related to the precession, main nutation, tilt-over-mode, …

The traditional theory of the figure of the earth + PREM

➔ H≈1/308.8

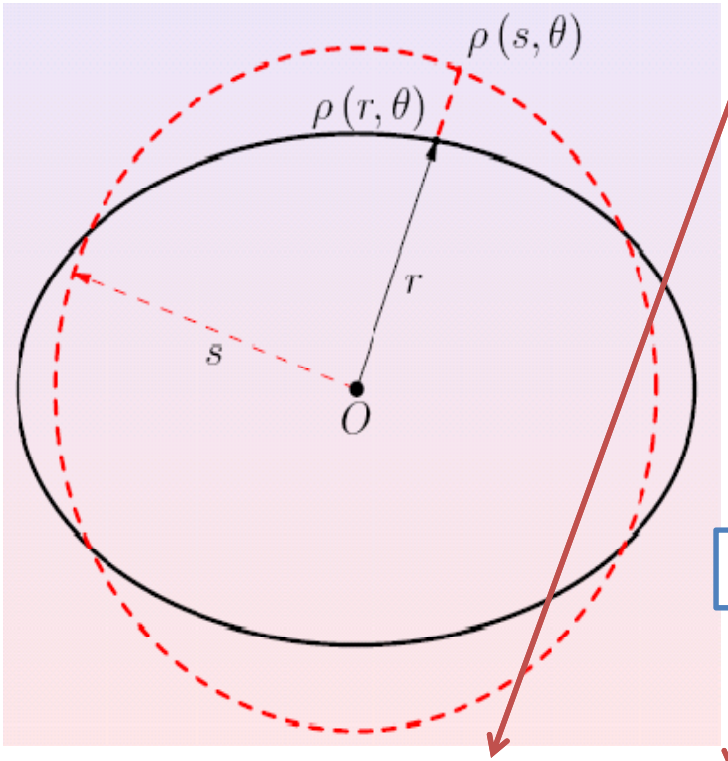
precise precession obs.

➔ H≈1/305.5

~1.1% difference

The report of IAU WG "nutation", 2000;
《The solved & unsolved problems in Earth rotation》

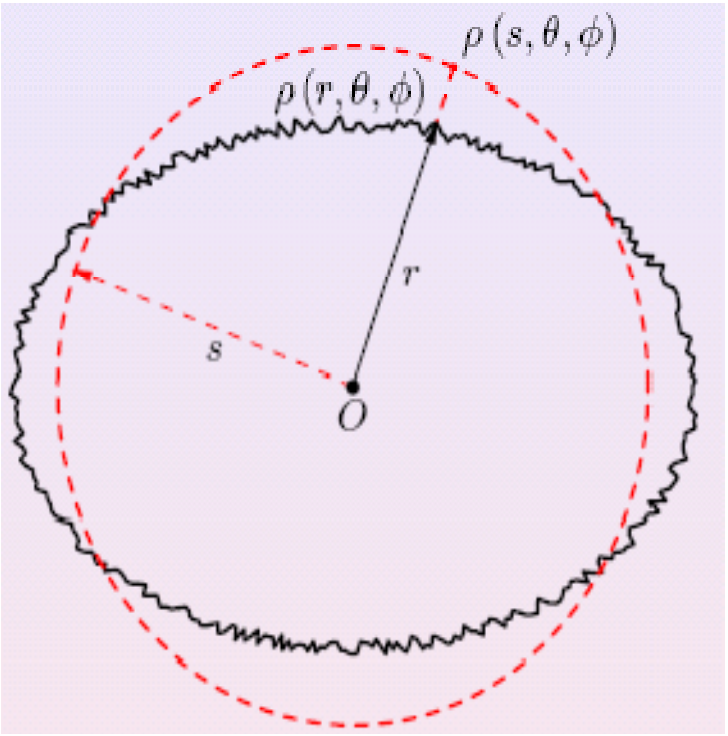
A generalized theory of figure of the earth



Clairaut,1743

Darwin,1899;
De Sitter,1924

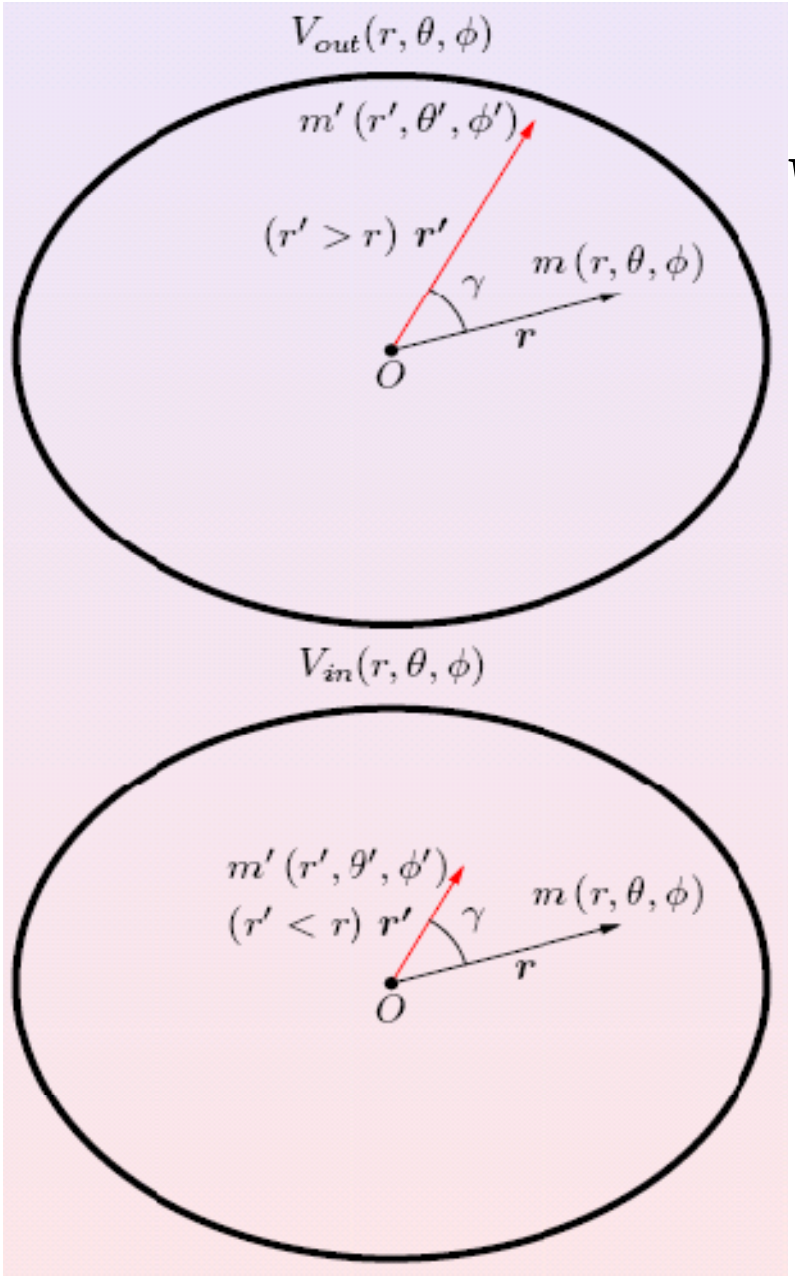
Denis,1985-2006



$$r = s\left[1 + s_2 P_2 + s_4 P_4 + s_6 P_6\right]$$

$$r(s,\theta,\phi) = s\left[1 + \sum_{n=0}^{\infty}\sum_{m=-n}^{n} H_n^m(s)\,Y_n^m(\theta,\phi)\right]$$

Calculation of gravity potential



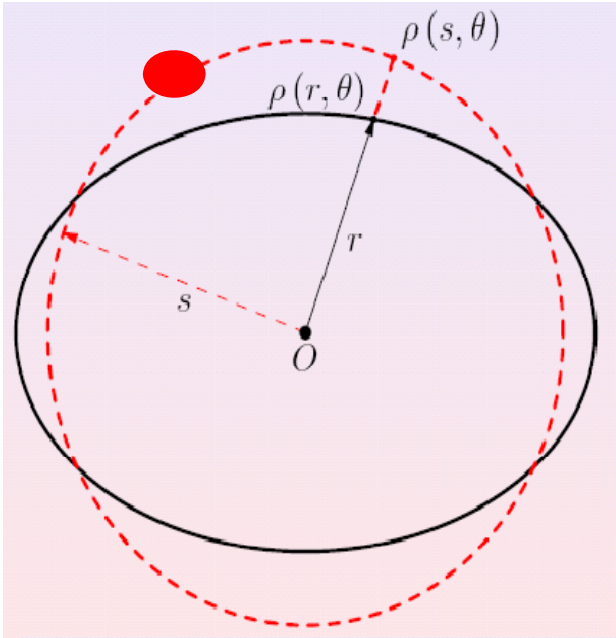
$$W(r,\theta,\phi)=V_{in}(r,\theta,\phi)+V_{out}(r,\theta,\phi)+Z(r,\theta,\phi)$$

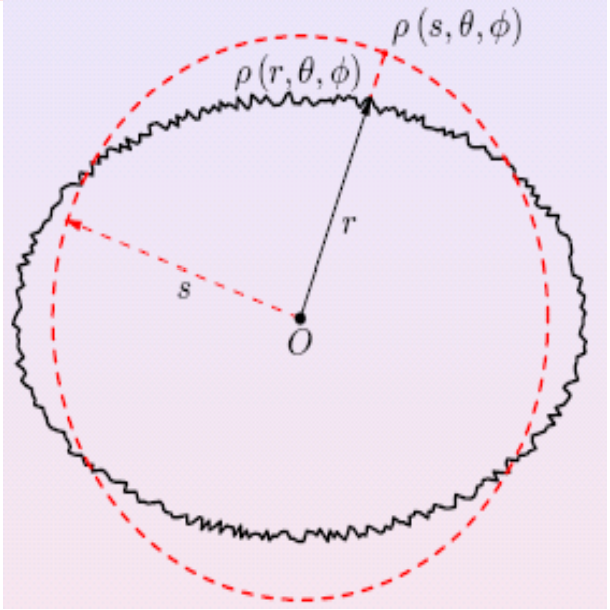
V_{in} / V_{out} : gravitational potential by the mass inside / outside the target equi-potential surface

Z: centrifugal potential

Direct effect

the crust inhomogeneous mass change directly the gravitational potential for all mass points interior in different ways, therefore, the figure of equi-potential surfaces interior are changed without symmetries.

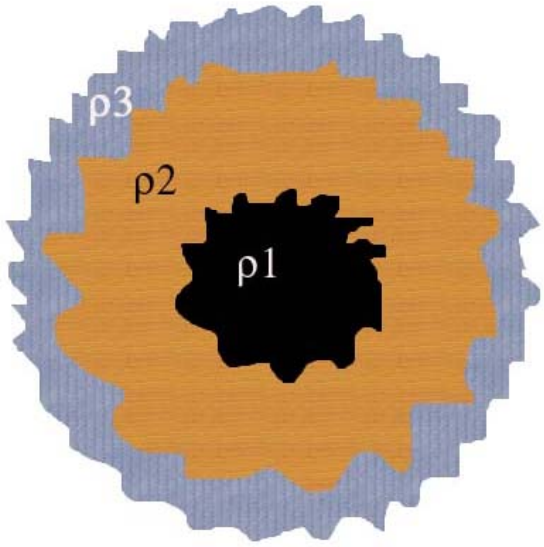


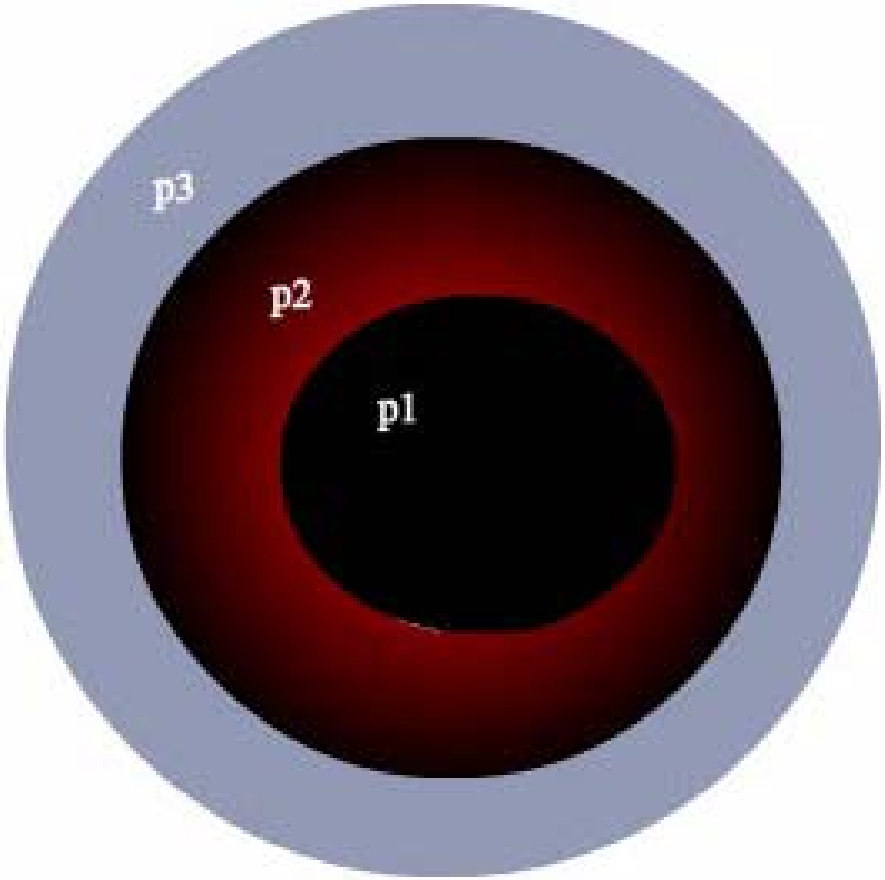


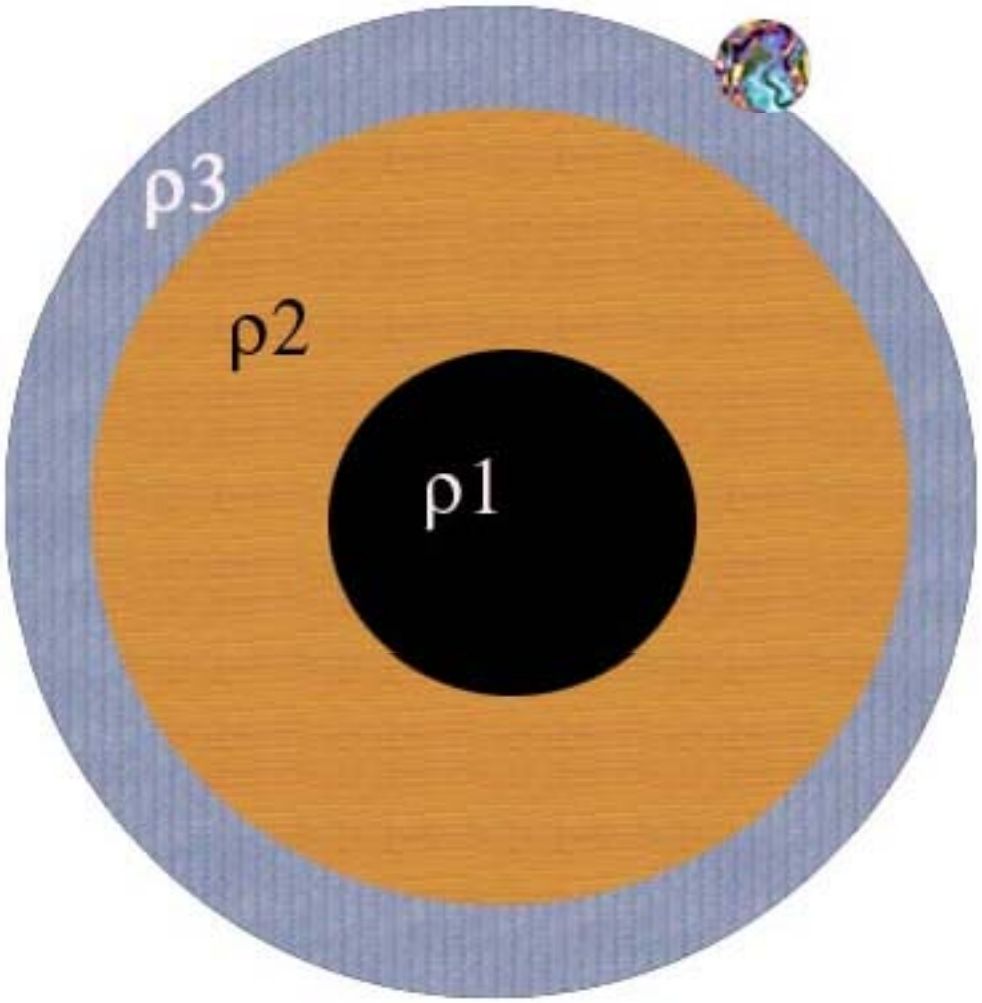
Indirect effect

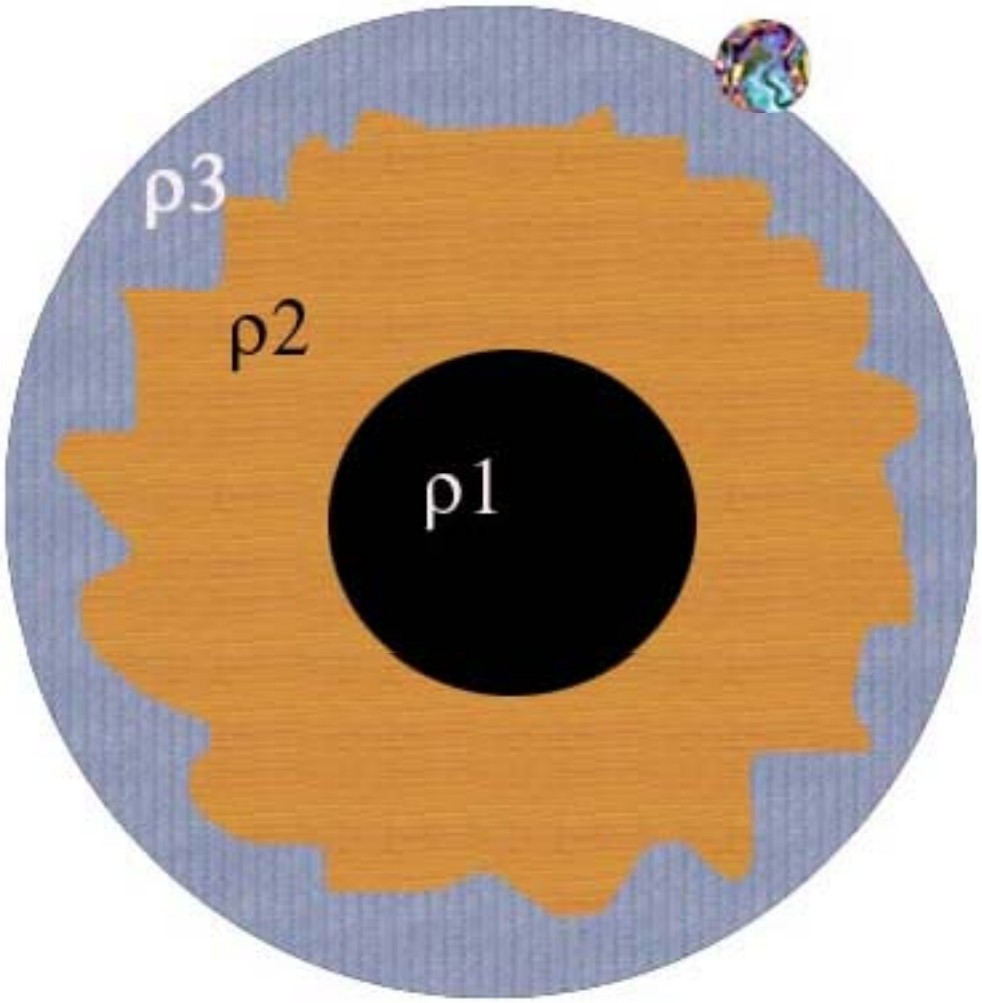
As the figures of equi-density surfaces (then the density distribution) interior are changed by the direct effect, the gravitational potential of other locations (outside/inside this surface) are changed, and the figures of equi-potential surfaces all through the earth are then changed again.

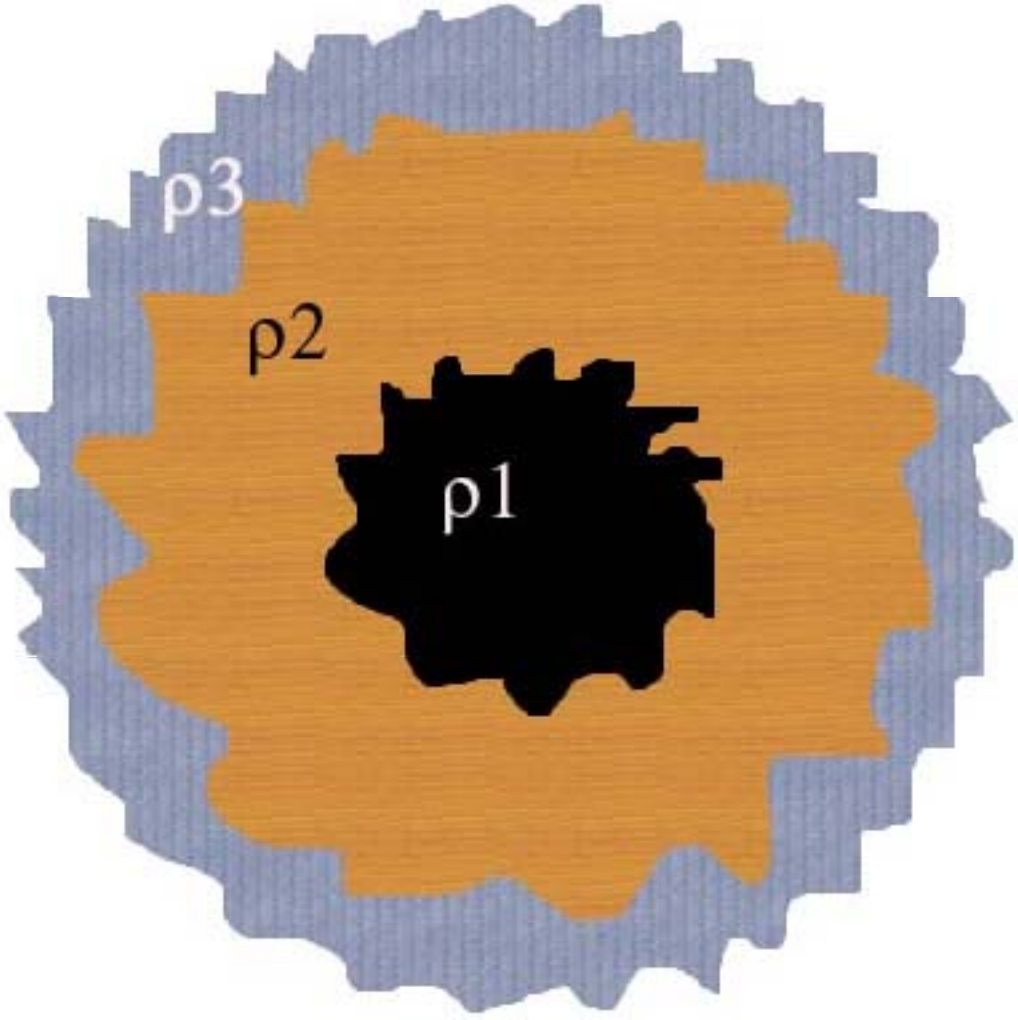
This process is reciprocal and needs iteration, and will finally reach equilibrium.




p3
p2
p1


ρ3
ρ2
ρ1


ρ3
ρ2
ρ1


ρ3
ρ2
ρ1

The final eqs. of the figure of the equi-potential surfaces

$$
\begin{aligned}
W &= V_{in} + V_{out} + Z \\
&= GE_0(s) + G\rho s^2 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m \left[m_h p_{n,m} + \sum_{l=0}^{\infty} \frac{s^{l-2}}{\bar{\rho}} u_{l,n,m} \right. \\
&\qquad \left. + \sum_{l=1}^{\infty} g_{l,n,m} + \sum_{l=0}^{\infty} f_{l,n,m} \right] \\
&= GE_0(s) + G\bar{\rho} s^2 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\theta, \phi) \Xi_n^m(s) \\
\Xi_n^m &= m_h p_{n,m} + \sum_{l=0}^{\infty} \frac{s^{l-2}}{\bar{\rho}} u_{l,n,m} + \sum_{l=1}^{\infty} g_{l,n,m} + \sum_{l=0}^{\infty} f_{l,n,m}
\end{aligned}
$$

$$
\begin{cases}
\Xi_n^m + (-1)^m \Xi_n^{-m*} = 0 \\
n = 1, \ldots, \infty \\
m = 0, \ldots, n
\end{cases}
$$

Strategy to solve the eqs.

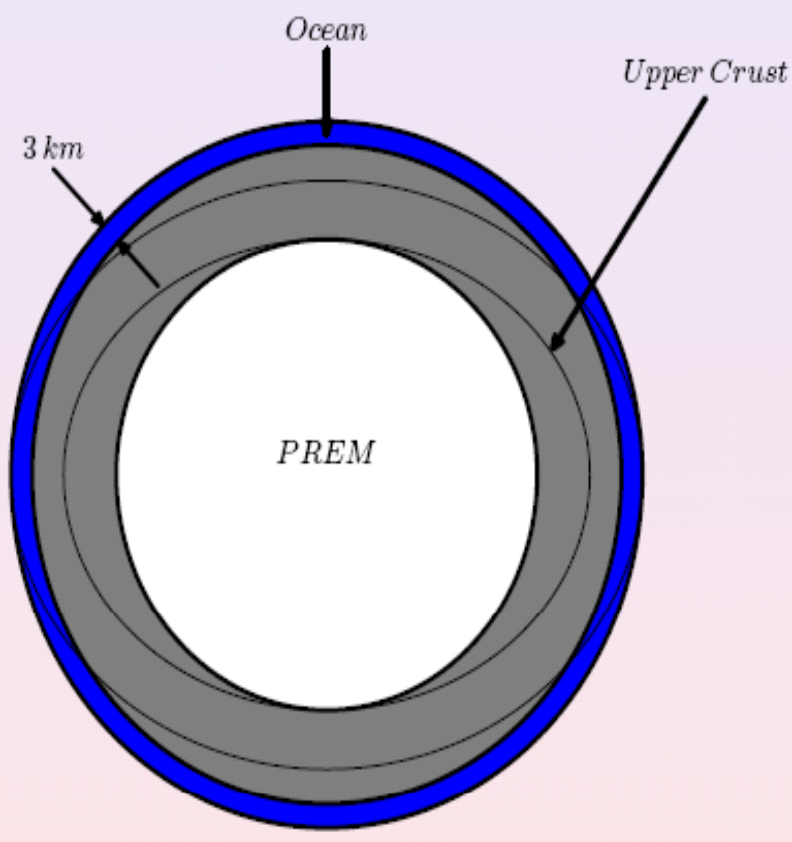
- **Truncated to n & m=6: 3-rd order precision**

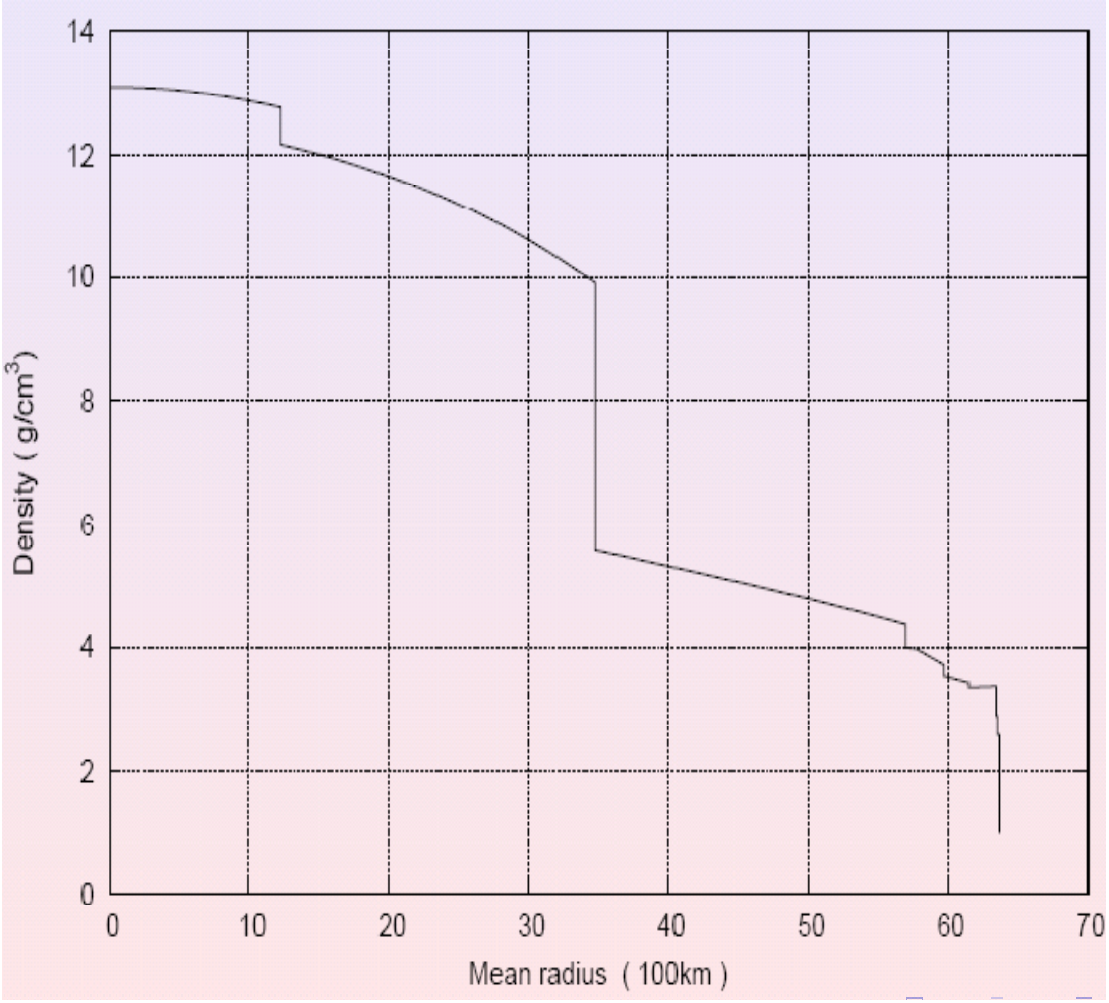
  truncated to 8: no obvious change at 3rd. order precision?

- **Detail derivation for the formulas of the hundrads parameters are done by symbol processing software "Mathmatica"**

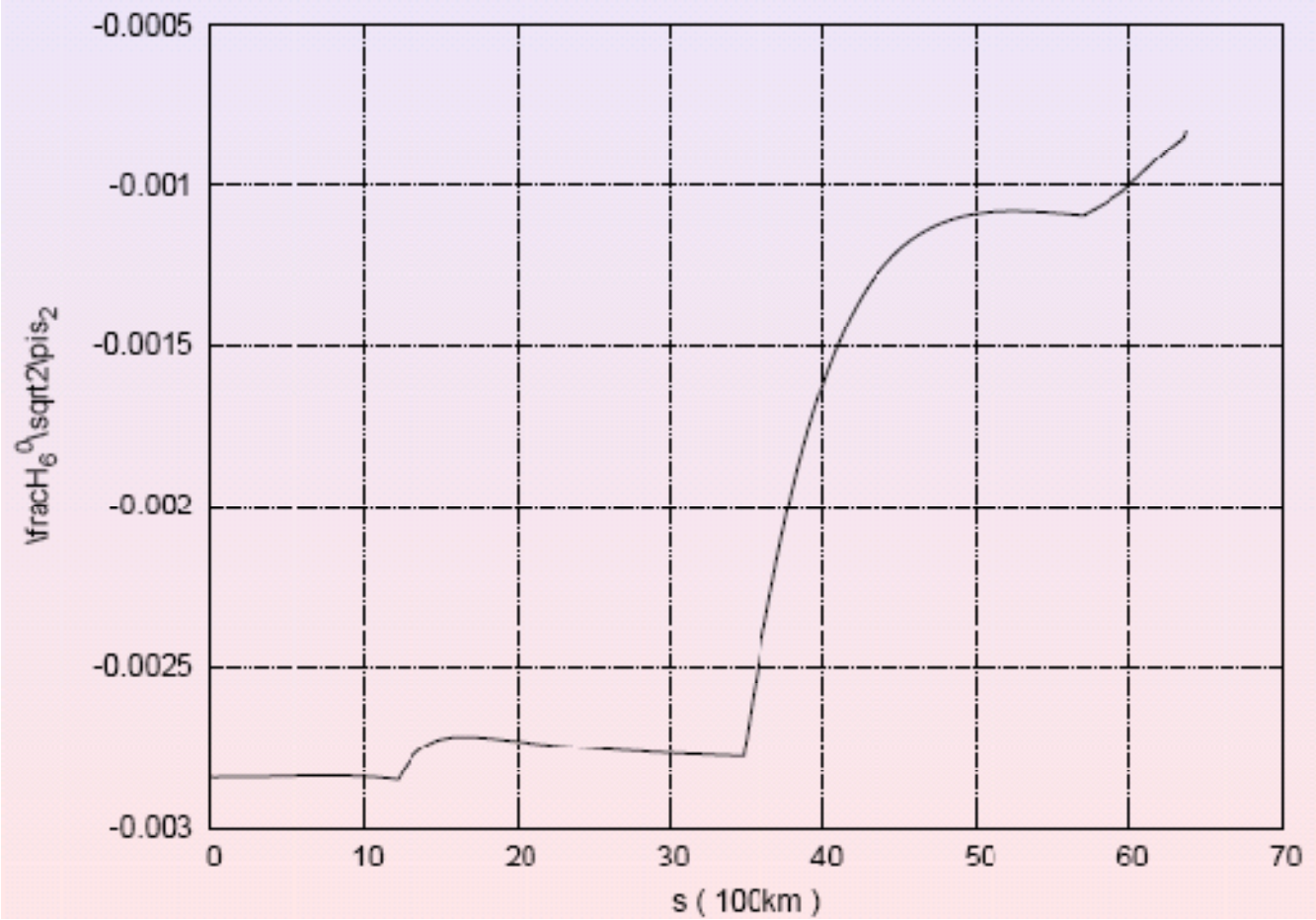
- **the parameters for real earth are then computed from the center to the outer surface in the self-consistent formulas by *iterations*.**

Earth model: PREM (Dziewonski & Anderson, 1981)





Validation of this new theory: degenerate to symmetric earth (PREM)



Agree with Denis(1989) very well.

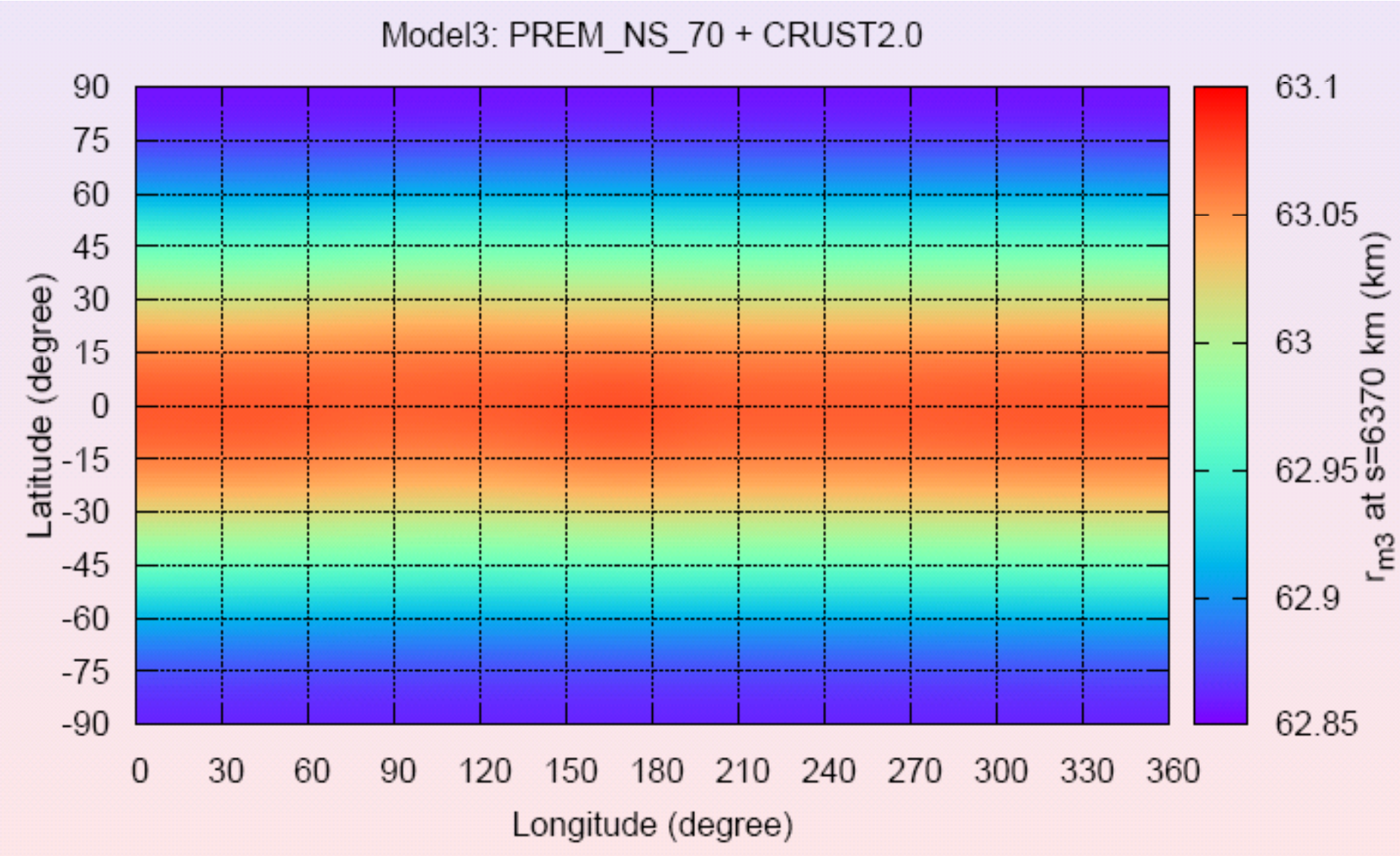
The relative diff. between H_6^0 & S_6 $\left(H_6^0(s)\sqrt{\frac{13}{4\pi}} - s_6(s)\right)/s_6(s)$

Models for the more real Earth

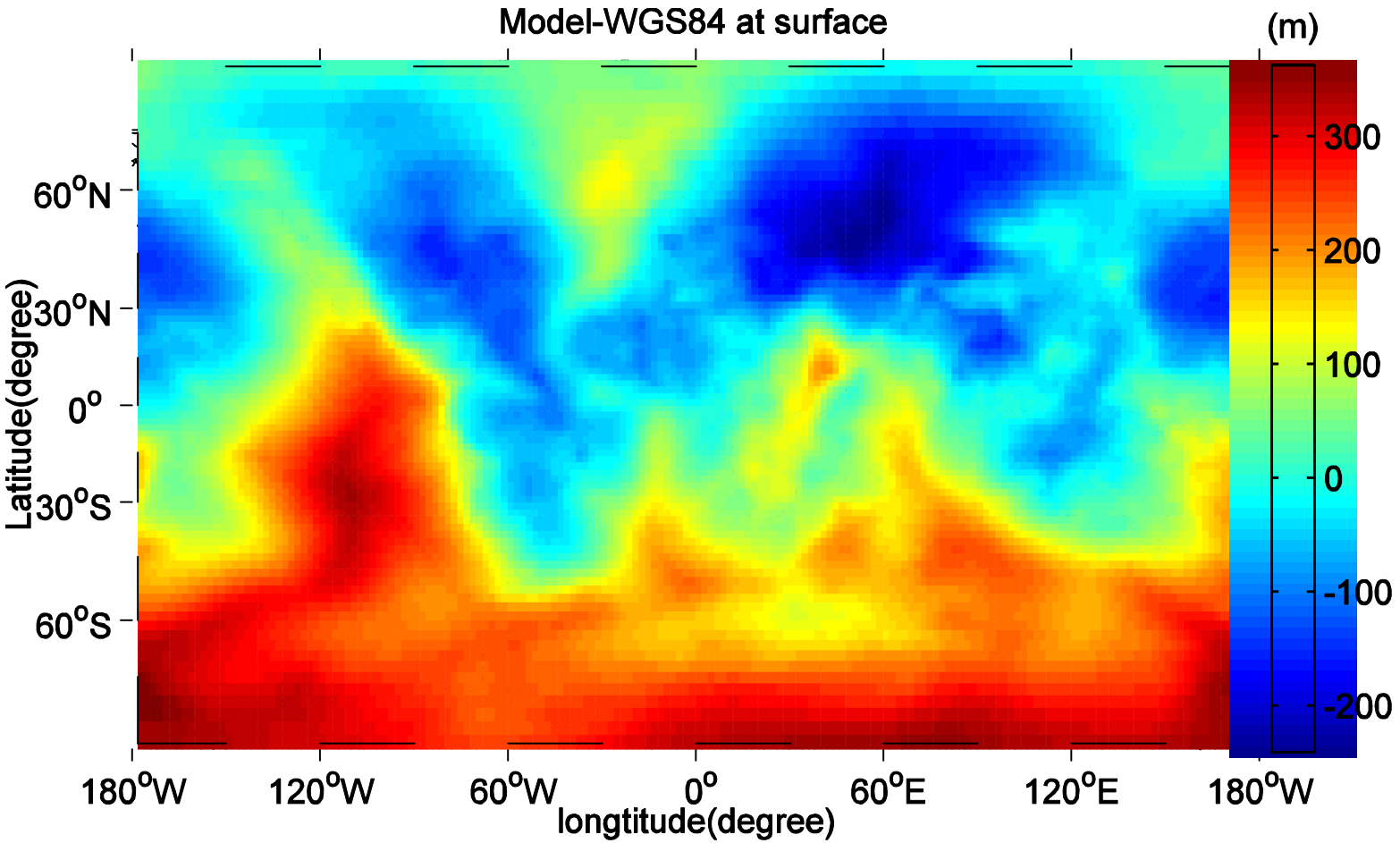
the ocean/topography models used here

	ECCO	GTOPO30	ETOPO5	CRUST2.0
Source	NOPP	USGS	NOAA	Chulick etc.
Layers no.	46	–	–	8
Depth(km)	5.615	0	10.376	70.137
Grid res.	不均匀	$30'' \times 30''$	$5' \times 5'$	$2^\circ \times 2^\circ$

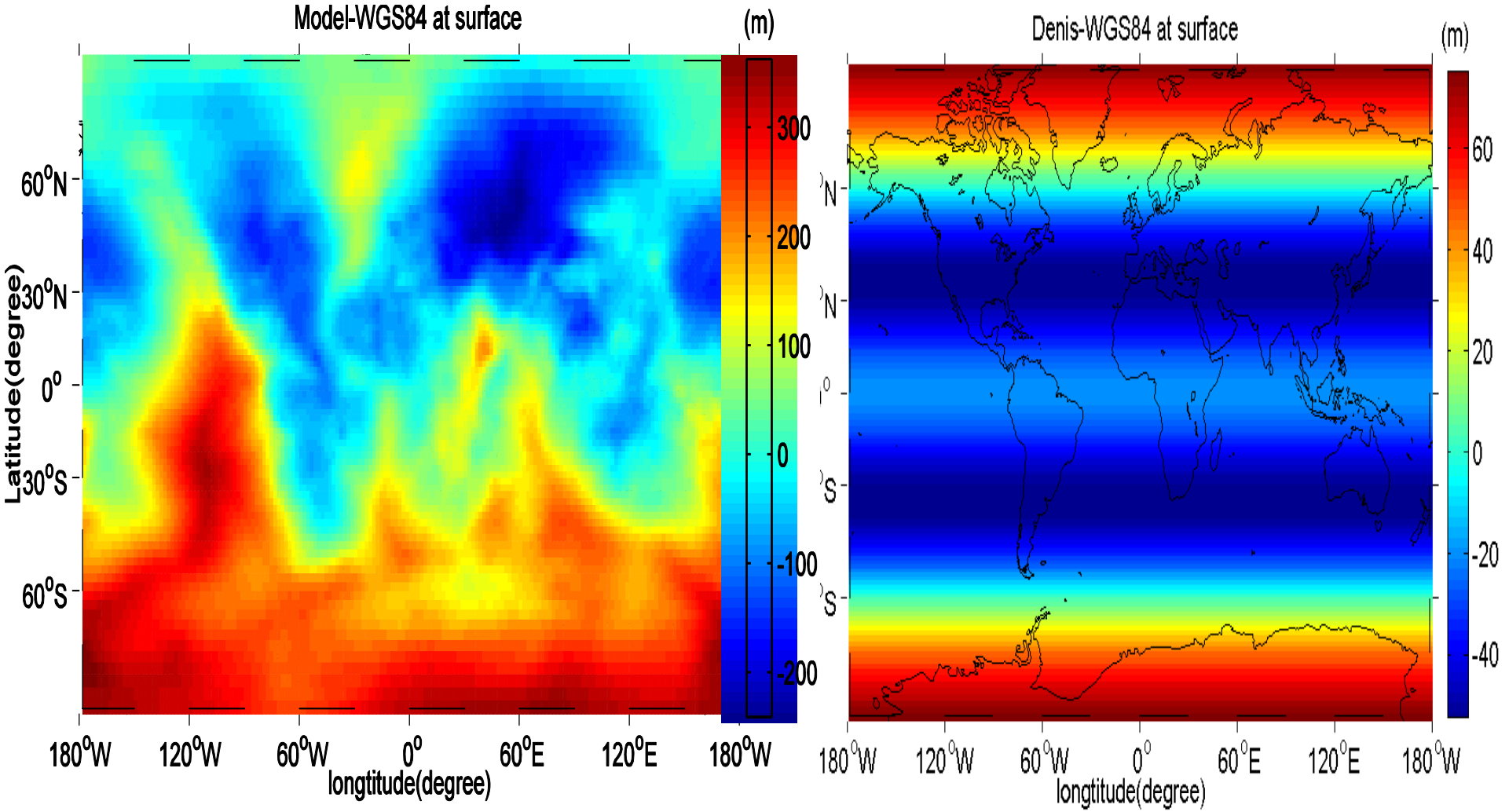
Results: the profiles of the interior geoid



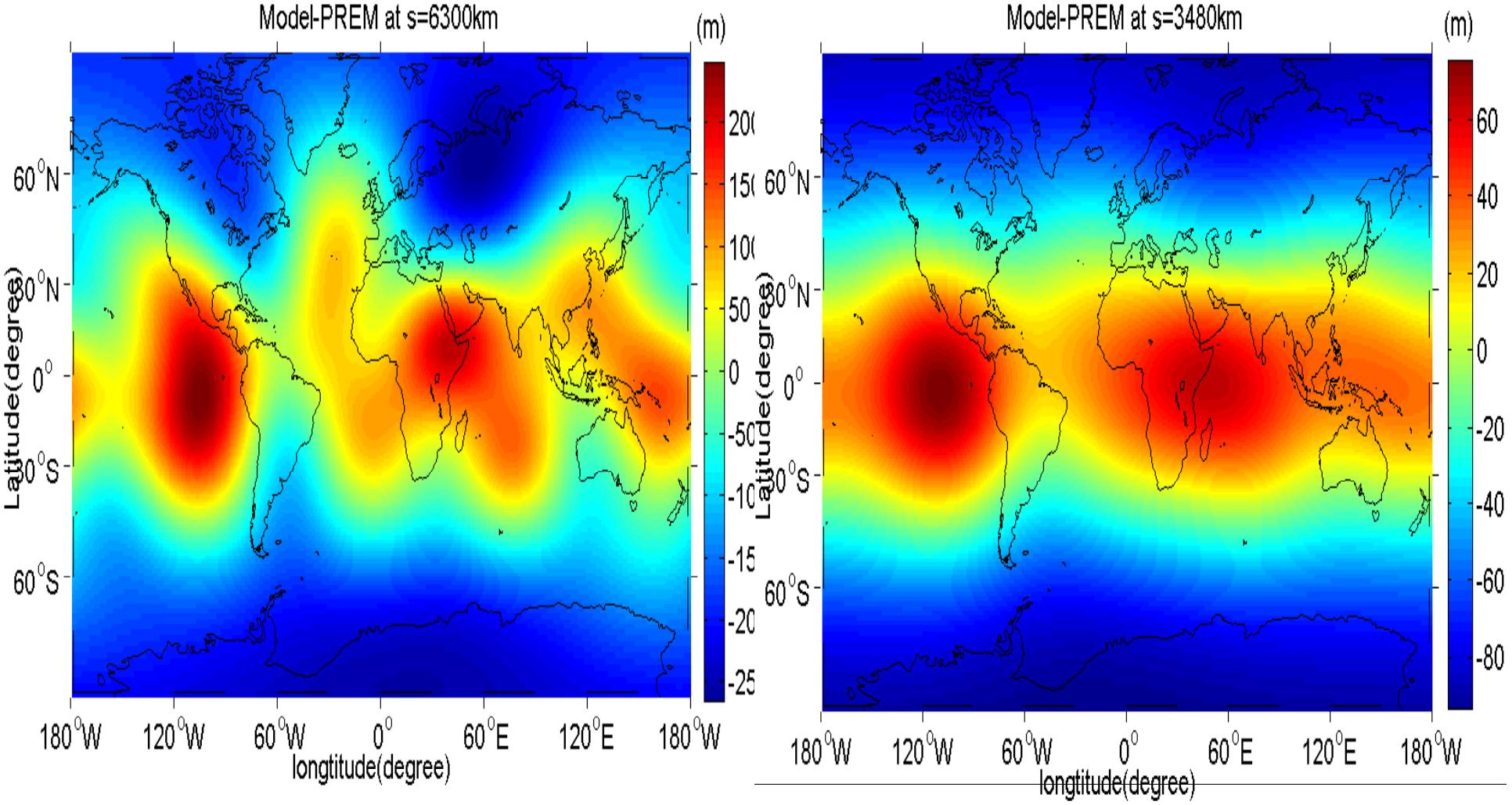
Results: the profiles of the interior geoid



Results: the profiles of the interior geoid



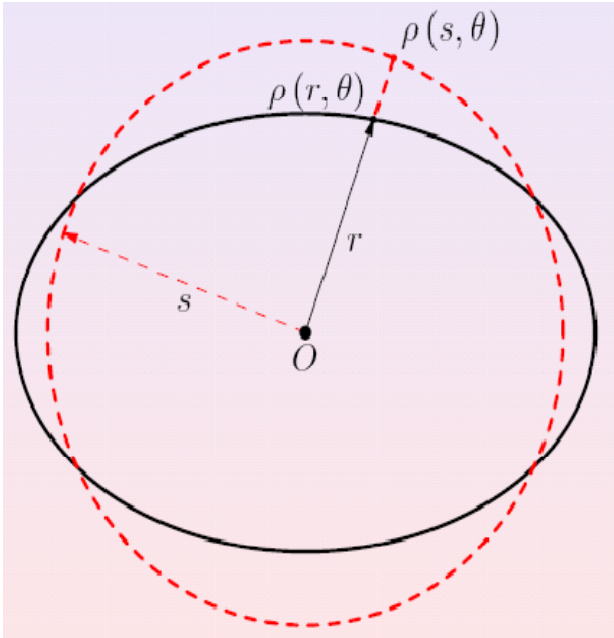
Results: the profiles of the interior geoid



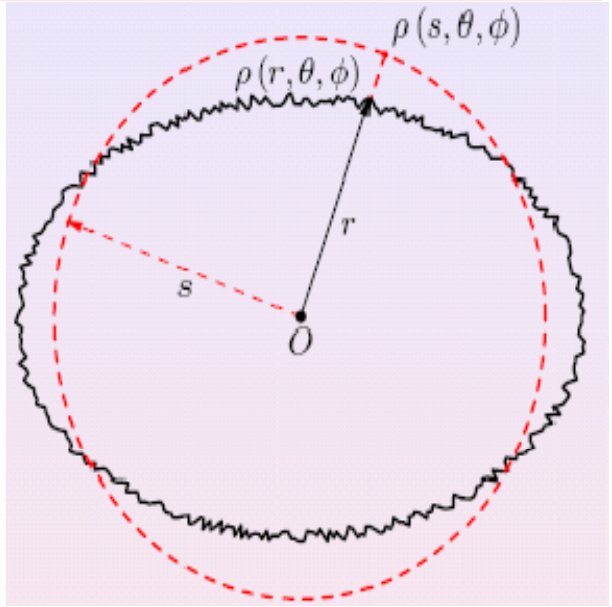
Results: H

$$H = \frac{C - \frac{A+B}{2}}{C}$$

$$C = \int_v (x^2 + y^2)\, \rho \, \mathrm{d}V$$



• Direct contribution of the topo. to H:
change the mass distribution in the integral (in the top layers only)



• Indirect contribution of the topo. to H:
change the figures of equi-density surfaces interior, then change the density distribution (all through the earth!)

Results: MoI & H

- Direct effect considered only

Obs.: 305.5

	A $10^{37}\ kg \cdot m^2$	B $10^{37}\ kg \cdot m^2$	C $10^{37}\ kg \cdot m^2$	1/H
PREM(-71km)	7.7087284	7.7087284	7.7336553	
CRUST2.0	0.2949340	0.2947971	0.2956929	
TOTAL	8.0036624	8.0035255	8.0293482	311.7674842

- Both direct & indirect effects

	A $10^{37}\ kg \cdot m^2$	B $10^{37}\ kg \cdot m^2$	C $10^{37}\ kg \cdot m^2$	1/H
PREM(All)	8.0112987	8.0112987	8.0373506	308.5131401
PREM(-71km)	7.7164775	7.7164823	7.7418221	
CRUST2.0	0.2948918	0.2947790	0.2957497	
TOTAL	8.0113693	8.0112612	8.0375718	306.1164533

+1%

+0.2%

Summary & remarks

1. **A new generalized integrated formula to obtain the equilibrium figures to 3^{rd}-order accuracy for real earth is developed. All the non-zero order and odd degree terms are included in the SH expression of the figures.**

2. **In these formulas, both the direct & indir. contributions of the anti-symmetric crust layers are included.**

3. **Profiles of the equilibrium figures, no longer symmetric, interior the real Earth are obtained; and comparison among them provides an indirect evidence & support for the theory of isostasy**

Summary & remarks

4. The calculated geoid embodies stronger topographical signal than that calculated by traditional theory.

5. The direct effect of the real ocean and topo. layers up to 70km depth changes H by ~0.7% in opposite direction; while the indirect effect, based on thiswork, can draw back the difference of H_{theory}-$H_{obs.}$ from 1% to 0.2%.


Thank You!