Influence of the inner core on the rotation of the Earth revisited

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Contents

External gravitational potential energy

- External celestial bodies interact gravitationally with the non–spherical Earth, modifying its rotational evolution
- Those interactions depend on the gravitational potential $\mathcal{V}$
- The influence of the Earth model on its rotational motion can be explained more clearly by casting the potential $\mathcal{V}$ in two parts as

$$\mathcal{V} = \mathcal{V}_{\text{ref}} + \Delta\mathcal{V}$$

  - ✓ The reference part, $\mathcal{V}_{\text{ref}}$, is the potential energy of the Earth associated to a chosen configuration of reference- commonly, that used in rigid Earth theory
  - ✓ The redistribution part, $\Delta\mathcal{V}$, provides the potential energy emerging from the differences with respect to the reference configuration. It depends on the nature of the Earth model under consideration, besides the disturbing bodies

Reference contributions

- The main part to the precession and the long period nutations comes from the second degree harmonic

$$\mathcal{V}_{\text{ref}} = G\frac{m}{r^3}eAC_{20}\left(\eta,\alpha\right), \text{ with } e = \frac{C-A}{A},$$

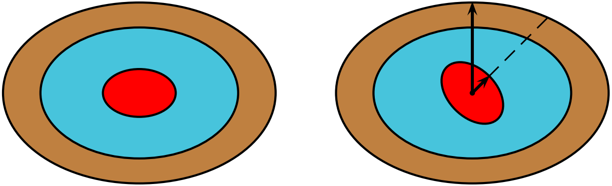
  r, η, and α being the distance, colatitude and longitude of a relevant perturbing body.
- Commonly Earth rotation studies focus on this main term by considering different one, two, or three layer Earth models
- Once a potential of reference $\mathcal{V}_{\text{ref}}$ is chosen, the solutions

$$\Delta\psi = \sum_i A_i\left(n_i;\overline{p}\right)\sin\Theta_i,\ \Delta\varepsilon = \sum_i B_i\left(n_i;\overline{p}\right)\cos\Theta_i,$$

  depend on the Earth model, due to the existence of different normal modes (CW, FCN, PFCN, ICW) for different models. Hence the corresponding transfer functions (or generating functions) are different

Redistribution contributions

- In this presentation we will call redistribution contributions to those associated to certain changes in the mass distribution inside the Earth
- They appear when considering either elastic or even simpler three layer Earth models
- That is because an actual rotation of the solid inner core (SIC) must change the inertia tensor of the Earth



Objectives

- In this talk we will report on the redistribution contributions to the motion of the Earth figure axis that emerge when considering unrestricted three layer Earth models, i.e. models in which the SIC is not constrained to move without changing the inertia tensor of the Earth
- Let us recall that the usual solutions consider a rather virtual motion of SIC, affecting the free oscillation frequencies but not the tensor of inertia of the Earth (gravitational potential), so that those effects are somehow indirect
- Although these direct effects are small, they can provide contributions to the Earth rotation motion reaching the level of some tens of micro arcseconds (μas)

Three layer Earth model

- We will consider an Earth model composed of three nearly spherical, ellipsoidal layers
  - ✓ An axial–symmetric rigid mantle
  - ✓ An stratified fluid outer core (FOC)
  - ✓ An axial–symmetric rigid inner core (SIC)
- The presence of the inner core forces to incorporate its differential rotation with respect to the other layers in the dynamics
- This internal rotation of the inner core, inherent to the model, produces a redistribution of mass that originates a change $\Delta\mathcal{V}$ in the gravitational potential energy
- Should that change be neglected, the full direct and indirect effects of SIC would not be accounted for totally

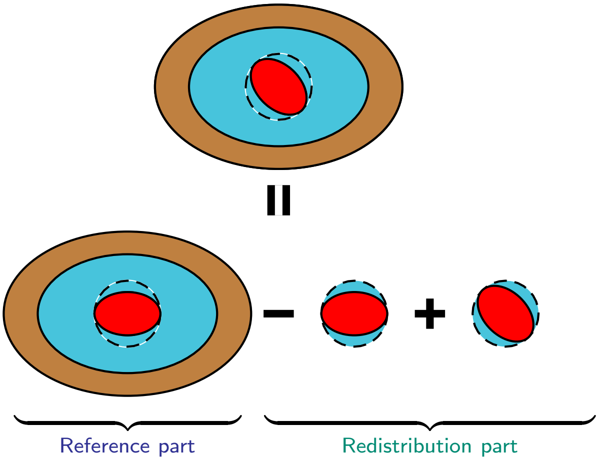
Expression of the gravitational potential energy

- Second degree harmonic part of the geopotential can be derived from MacCullagh's formula

$$\mathcal{V} = G\frac{m}{2r^5}\left[3\begin{pmatrix} x \\ y \\ z \end{pmatrix}^t \Pi \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \text{trace}\,(\Pi)\; r^2\right]$$

- Therefore, to obtain both the reference and the redistribution parts of the potential energy it is necessary to express the matrix of inertia Π of the three layer Earth with respect to a mantle fixed system

Matrix of inertia of the Earth



Expression of the reference potential and its change by mass redistribution

- In the mantle system the matrix of inertia of the Earth is

$$\Pi = A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+e \end{pmatrix} + A_s (e_s - \delta) \begin{pmatrix} k_1^2 & k_1 k_2 & k_1 k_3 \\ k_1 k_2 & k_2^2 & k_2 k_3 \\ k_1 k_3 & k_2 k_3 & k_3^2 - 1 \end{pmatrix},$$

  k_i being the components of the SIC figure axis unit vector

- Therefore, the gravitational potential energy is given by

$$\mathcal{V}_{\text{ref}} = G\frac{m}{r^3} A e C_{20}(\eta, \alpha),$$

$$\Delta\mathcal{V} = G\frac{m}{r^3} A_s (e_s - \delta) \left[\frac{2\left(k_3^2 - 1\right) - k_1^2 - k_2^2}{2} C_{20}(\eta, \alpha) + k_1 k_3 C_{21}(\eta, \alpha) + \right.$$

$$+ \left. k_2 k_3 S_{21}(\eta, \alpha) + \frac{k_1^2 - k_2^2}{4} C_{22}(\eta, \alpha) + \frac{k_1 k_2}{2} S_{22}(\eta, \alpha) \right].$$

Analytical solution

- The equations of motion are derived and solved by means of the Hamiltonian formalism (Getino & Ferrándiz 2001)
- The Hamiltonian is expressed in an Andoyer–like set of variables (e.g., Escapa et al. 2001) and the solution is obtained by perturbation methods (Hori 1966)

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$$

- The unperturbed part $\mathcal{H}_0$ accounts for the hydrodynamical and internal gravitational interactions, and the perturbation is given by

$$\mathcal{H}_1 = \mathcal{V}_{\text{ref}} + \Delta\mathcal{V}$$

- This procedure allows us to isolate both the solutions corresponding to the reference potential and to the redistribution contributions (at the first order) and to obtain their analytical expression (Escapa et al. 2012)

Figure axis redistribution contributions

- We have obtained a preliminary quantification of the redistribution contributions

Argument					Period	Figure axis (μas)	
l_M	l_S	F	D	$\overline{\Omega}$	Days	$\Delta\psi$	$\Delta\varepsilon$
+0	+0	+0	+0	+1	-6793.48	2.79	-0.31
+0	+0	+0	+0	+2	-3396.74	0.00	-0.01
+0	+1	+0	+0	+0	365.26	14.95	9.29
+0	-1	+2	-2	+2	365.25	-1.78	0.48
+0	+0	+2	-2	+2	182.63	44.61	-19.92
+0	+1	+2	-2	+2	121.75	1.64	-0.72
+1	+0	+0	+0	+0	27.55	-2.22	0.02
+0	+0	+2	+0	+2	13.66	7.17	-3.08
+0	+0	+2	+0	+1	13.63	1.22	-0.63
+1	+0	+2	+0	+2	9.13	0.96	-0.41

Summary I

- Even in the non–elastic situation, three layer Earth models provides non negligeable contributions to the Earth rotation due to the mass redistribution caused by the inner core rotation, besides their indirect effect through the two normal modes associated to it
- That redistribution of masses inside the Earth modifies the gravitational potential energy of the Earth attracted by a system of external bodies.
- With the help of the Hamiltonian formalism, we have derived analytical expressions for those redistribution contributions on the motion of the figure axis

Summary II

- Preliminary values of the amplitudes of that direct, redistribution contributions show that some terms reach the magnitude of some tens μas

$$\Delta\psi = 14.9\sin(l') + 44.61\sin(2F - 2D + 2\Omega) + 7.17\sin(2F + 2\Omega),$$
$$\Delta\varepsilon = 9.29\cos(l') - 19.92\cos(2F - 2D + 2\Omega) - 3.08\cos(2F + 2\Omega),$$

  that is to say, they are of the same order of other effects currently considered or investigated
- To our knowledge those contributions have not been taken into account up to this date. In fact, the available solutions to three layer Earth models consider a fixed luni–solar potential (the reference potential here), the same as in the underlying rigid Earth theory to which a transfer function is applied.
- In view of its magnitude those effects should be considered for inclusion in the actual standards and models

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