Evaluation of the accuracy of the IAU 2006/2000 precession-nutation

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The IAU 2000/2006 definitions and models for precession-nutation

IAU 2000 Resolutions

Resolution B1.3
Definition of BCRS and GCRS

Resolution B1.6
IAU 2000 Precession-Nutation Model

Resolution B1.7
Definition of Celestial Intermediate Pole

Resolution B1.8
Definition and use of CEO and TEO

IAU 2006 Resolutions

Resolution B1
Adoption of the P03 Precession and definition of the ecliptic

Resolution B2
Harmonization of the names to CIO and TIO
IAU 2006/2000A Precession-nutation

IAU 2000 (Resolution B1.6)
- adopted the IAU2000 precession-nutation (Mathews, Herring, Buffett, 2002) which was implemented in the IERS Conventions 2003

**IAU 2000A Nutation** (non-rigid Earth model)

**IAU 2000 Precession** = IAU 1976 (Lieske et al. 1977) + corrections to precession rates

\[ d\psi_A (\text{IAU 2000}) = -0.29965''/c \]
\[ d\omega_A (\text{IAU 2000}) = -0.02524''/c \]

*Celestial pole offsets* at J2000 (VLBI estimates)

\[ \xi_0 (\text{IAU 2000}) = -16.6170 \text{ mas} \]
\[ \eta_0 (\text{IAU 2000}) = -6.8192 \text{ mas} \]

IAU 2006 (Resolution B1)
- adopted the P03 precession (Capitaine, Chapront, Wallace, 2003) dynamical model consistent with IAU 2000A nutation and with non-rigid Earth which was implemented in the IERS Conventions 2010
- recommended improved definitions (ecliptic, precession of the equator, precession of the ecliptic)
Main features of the IAU 2000 nutation

MHB nutation for a non-rigid Earth: Rigid Earth nutation (prograde and retrograde amplitudes) * transfer function

*Rigid Earth nutation* (Souchay et al. 1999)
Analytical solution providing semi-analytical series: 1365 luni-solar and planetary terms “in-phase” and “out-of-phase” components (amplitudes between 17.2” and 0.1 µas ; periods between 3 d and 101 cy).

*transfer function* (Mathews et al. 2002)
Derived from the solution of equations obtained by generalization of the *SOS equations* (Sasao et al. 1980) for the variations in rotation of the Earth’s mantle and fluid core, with *Basic Earth Parameters (BEP)* based on *Model for the dynamics of the Earth’s interior and for modeling the dissipative phenomena*) and fitted to VLBI data.

- $e, e_f$: dynamical ellipticity of the Earth and its fluid core, respectively,
- $\kappa = e_k^2/k_s$, $\gamma$: compliance parameters representing the deformabilities of the whole Earth and its fluid core, respectively under tidal forcing,
- $K^{CMB}$ and $K^{ICB}$: core-mantle and outer core to inner core couplings due to the magnetic fields crossing the boundaries of the fluid core,

The scale factor for the precession rate and nutation amplitudes is $S_{MHB} = H_d = e/(1 + e)$.

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<th>Basic Earth Parameters</th>
<th>Estimate</th>
<th>Correction to hydrostatic equilibrium</th>
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MHB BEP estimated from VLBI
(Mathews et al. 2002)
Main features of the IAU 2006 precession

• The IAU 2006 precession provides improved polynomial expressions for both the precession of the ecliptic and the precession of the equator, the latter being consistent with dynamical theory while matching the IAU 2000A precession rate for continuity reasons.

• The precession of the equator was derived from the dynamical equation expressing the motion of the mean pole about the ecliptic pole.

• The solution is based on:
  – the IAU 2000 precession rates in longitude and obliquity,
  – the value, \( \varepsilon_0 = 84381.406'' \), from Chapront et al. (2002) for the mean obliquity of the ecliptic at J2000.0,
  – contributions to the precession rates \( r_\psi \), \( r_\varepsilon \) from Williams 1994, Brumberg et al. 1998, Mathews et al. 2002,
  – correction in the precession rate for the change in the J2000 obliquity from IAU2000 to P03,
  – \( \frac{dJ_2}{dt} = - 3.0 \times 10^{-11}/ \text{yr} \)
IAU 2006 expressions for precession

(Capitaine et al. 2003)

\( \psi_{A1} \times \sin \varepsilon, \omega_{A1} \) : components of the precession rates of the equator

\( (X_1, Y_1) \) : components of the precession rates of the equator

The polynomial coefficients for all the precession angles are in Hilton et al. (2006)
IAU 2006/2000A_{R06} expressions for the GCRS coordinates of the CIP

\[ X = -0.016617 + 2004.191898 \, t - 0.4297829 \, t^2 \\
- 0.19861834 \, t^3 - 0.00007578 \, t^4 + 0.000059285 \, t^5 \\
+ \sum_i [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{ARGUMENT})] \\
+ \sum_i [(a_{s,1})_i \, t \sin(\text{ARGUMENT}) + (a_{c,1})_i \, t \cos(\text{ARGUMENT})] \\
+ \sum_i [(a_{s,2})_i \, t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i \, t^2 \cos(\text{ARGUMENT})] \\
+ ... \]

\[ Y = -0.006951 - 0.025896 \, t - 22.4072747 \, t^2 \\
+ 0.00190059 \, t^3 + 0.01112526 \, t^4 + 0.0000001358 \, t^5 \\
+ \sum_i [(b_{c,0})_i \cos(\text{ARGUMENT}) + (b_{s,0})_i \sin(\text{ARGUMENT})] \\
+ \sum_i [(b_{c,1})_i \, t \cos(\text{ARGUMENT}) + (b_{s,1})_i \, t \sin(\text{ARGUMENT})] \\
+ \sum_i [(b_{c,2})_i \, t^2 \cos(\text{ARGUMENT}) + (b_{s,2})_i \, t^2 \sin(\text{ARGUMENT})] \\
+ ... \]

\textit{precession; effects of frame biases; nutation; cross terms precession} \times \textit{nutation} \\
(a_{s,1})_i, (a_{c,1})_i, (b_{c,1})_i, (b_{s,1})_i : \text{take into account the } dJ_2/dt \text{ contribution}
Recently proposed improvements in precession-nutation theory

• Consideration of a relativistic theory of precession and nutation: effect of (i) the post-Newtonian torque, (ii) the geodesic precession as an additional torque (Klioner et al., Proc. Journées 2007).

• Influence of the inner core geopotential variations on nutation (Escapa et al., Proc. Journées 2010).


• Contribution of the Poisson terms of the tidal potential to nutation (Folgueira et al. A&A 469, 2007).


• Effect of the physical properties of the core-mantle boundary (Koot et al., Proc Journées 2007 and 2011).

• Precession expressions for long time intervals (Vondrák et al., A&A 2011).

→ Effects of amplitudes from a tens of μas to a ten of μas in the periodic terms and hundreds of μas/cy for Poisson terms and linear terms
The observed precession-nutation: VLBI celestial pole offsets
(corrections to the IAU 2006/2000 precession-nutation model)

Current uncertainty
~ a few tens of µas for X, Y

VLBI celestial pole offsets (w.r.t. the IAU 2006/2000 precession-nutation) corrected for the FCN

VLBI fit (secular and long period terms)

Data file opa2012a.eops contains 5138 records
First/last epoch 1979.59067520876 2012.56884845996

PARABOLA
========
-- X -------- -- Y --------
Prefit wrms: 0.147 0.149 mas
t^0 0.022 + 0.002 -0.086 + 0.002 mas
t^1 0.252 + 0.022 -0.436 + 0.023 mas/cy
t^2 2.140 + 0.266 0.418 + 0.271 mas/cy^2
Postfit wrms: 0.145 0.147 mas

Correlations:
t^0X 1.0
t^0Y 0.0 1.0
t^1X -0.1 0.0 1.0
t^1Y 0.0 -0.1 0.0 1.0
t^2X -0.6 0.0 -0.6 0.0 1.0
t^2Y 0.0 -0.5 0.0 -0.6 0.0 1.0
t^0X t^0Y t^1X t^1Y t^2X t^2Y

SLOPE + 18.6-yr
==========
-- X -------- -- Y --------
Prefit wrms: 0.147 0.149 mas
t^0 0.029 + 0.001 -0.086 + 0.001 mas
t^1 -0.008 + 0.021 -0.064 + 0.021 mas/cy
-- Real ------ -- Imag ------
ret 18.6 0.044 + 0.001 -0.022 + 0.001 mas
pro 18.6 0.025 + 0.001 -0.038 + 0.001 mas
-- X -------- -- Y --------
Postfit wrms: 0.140 0.142 mas

Correlations:
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t^0Y 0.0 1.0
t^1X 0.5 0.0 1.0
t^1Y 0.0 0.0 1.0 1.0
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R18im 0.0 0.0 -0.3 -0.3 0.0 1.0
P18re 0.0 0.1 -0.3 -0.3 0.1 0.2 1.0
P18im 0.0 0.0 0.0 0.3 -0.3 -0.2 0.1 0.0 1.0
t^0X t^0Y t^1X t^1Y R18re R18im P18re P18im

## Nutation corrections fit to VLBI

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### long periods

### short periods

Use of optical observations for secular terms

Celestial pole offsets (from Vondrák 2012)
Use of LLR observations for long period terms


JD7 Space-time reference systems for future research, 28th IAU GA, Beijing, August 2012
Use of GNSS observations for short period terms

Figure 8. Spectrum of circular nutation amplitudes (see (40)) at low periods generated from (a) the N3 series of GPS nutation rates relative to the IAU80 model, converted to actual nutation amplitudes using (37), and (b) the differences between the IERS96 and the IAU80 model. The dashed lines indicate the 1σ uncertainties of the amplitudes as expected according to (33) and (40).

Rothacher et al., JGR 1999, 104, 4835; also: Weber et al., 2001

Unongoing work on « Nutation determination using the Global Positioning System » by Yao et al 2012: poster JD7-3-1372 project in cooperation with Vienna TU (R. Weber, E. Umnig)

GINS and Bernese software
Conclusions

- The IAU 2006/2000 precession/nutation has been shown to be accurate up to a few hundreds of µas/cy in the linear term, a few tens of µas for the 18.6-yr nutation and better than about 15 µas for the other terms.

- However, this results from VLBI comparisons only. Other techniques should be used for check. For the moment no one can provide additional information, but studies are being done for improving the situation.

- The discrepancies of the IAU model are at the limit of what further theoretical computations predict; It is not possible to discriminate between several predictions yet.